Problem 2.11

Draw a field-line diagram for a uniformly charged solid sphere.

Solution

The electric field for a uniformly charged solid sphere was found in Problem 2.8.

$$\mathbf{E}(r) = \begin{cases} \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{R^3} r\hat{\mathbf{r}} & \text{if } r < R \\ \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}\hat{\mathbf{r}} & \text{if } r > R \end{cases}$$

Rewrite the electric field in Cartesian coordinates and in terms of dimensionless variables in order to graph it.

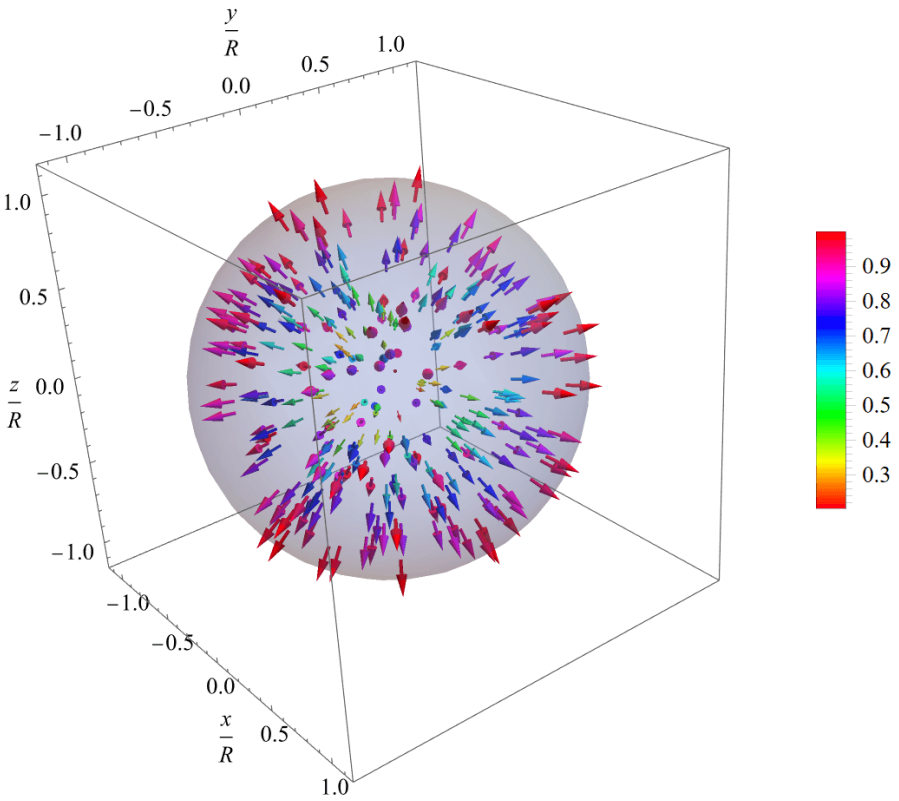
$$\frac{4\pi\epsilon_0 R^2}{q}\mathbf{E}(r) = \begin{cases} \dfrac{r}{R}\left(\dfrac{\mathbf{r}}{r}\right) & \text{if } \dfrac{r}{R} < 1 \\ \dfrac{R^2}{r^2}\left(\dfrac{\mathbf{r}}{r}\right) & \text{if } \dfrac{r}{R} > 1 \end{cases}$$

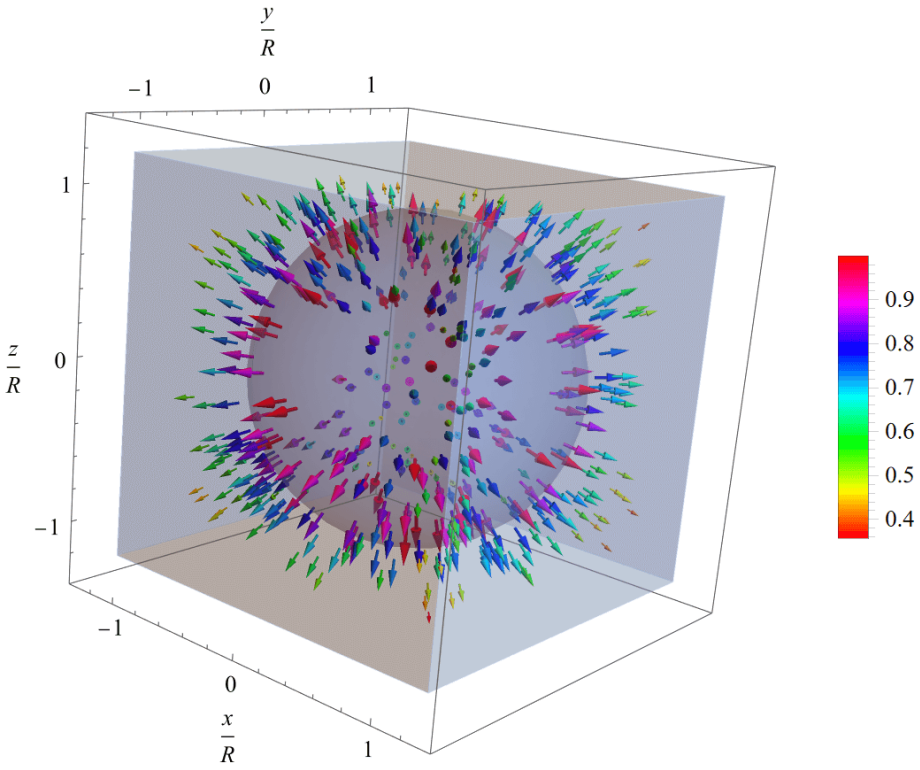
$$\frac{4\pi\epsilon_0 R^2}{q}\mathbf{E}(x,y,z) = \begin{cases} \dfrac{1}{R}\langle x, y, z\rangle & \text{if } \dfrac{1}{R}\sqrt{x^2+y^2+z^2} < 1 \\ \dfrac{1}{\frac{1}{R^2}(x^2+y^2+z^2)}\left(\dfrac{\frac{1}{R}\langle x,y,z\rangle}{\frac{1}{R}\sqrt{x^2+y^2+z^2}}\right) & \text{if } \dfrac{1}{R}\sqrt{x^2+y^2+z^2} > 1 \end{cases}$$

$$= \begin{cases} \left\langle \dfrac{x}{R}, \dfrac{y}{R}, \dfrac{z}{R}\right\rangle & \text{if } \sqrt{\dfrac{1}{R^2}(x^2+y^2+z^2)} < 1 \\ \dfrac{1}{\frac{x^2}{R^2}+\frac{y^2}{R^2}+\frac{z^2}{R^2}}\left[\dfrac{\left\langle \frac{x}{R}, \frac{y}{R}, \frac{z}{R}\right\rangle}{\sqrt{\frac{1}{R^2}(x^2+y^2+z^2)}}\right] & \text{if } \sqrt{\dfrac{1}{R^2}(x^2+y^2+z^2)} > 1 \end{cases}$$

$$= \begin{cases} \left\langle \dfrac{x}{R}, \dfrac{y}{R}, \dfrac{z}{R}\right\rangle & \text{if } \sqrt{\left(\dfrac{x}{R}\right)^2+\left(\dfrac{y}{R}\right)^2+\left(\dfrac{z}{R}\right)^2} < 1 \\ \left[\left(\dfrac{x}{R}\right)^2+\left(\dfrac{y}{R}\right)^2+\left(\dfrac{z}{R}\right)^2\right]^{-3/2}\left\langle \dfrac{x}{R}, \dfrac{y}{R}, \dfrac{z}{R}\right\rangle & \text{if } \sqrt{\left(\dfrac{x}{R}\right)^2+\left(\dfrac{y}{R}\right)^2+\left(\dfrac{z}{R}\right)^2} > 1 \end{cases}$$

On the next few pages are 3D plots of the electric field, both inside and outside the ball. The color and length of an arrow indicate the field's strength.


y/R
1.0
0.5
0.0
−0.5
−1.0
1.0
0.5
z/R 0.0
−0.5
−1.0
−1.0
−0.5
0.0
x/R
0.5
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3


$\frac{y}{R}$
−1
0
1
1
$\frac{z}{R}$
0
−1
−1
0
$\frac{x}{R}$
1
0.9
0.8
0.7
0.6
0.5
0.4

Below the vector fields inside and outside the ball are shown.